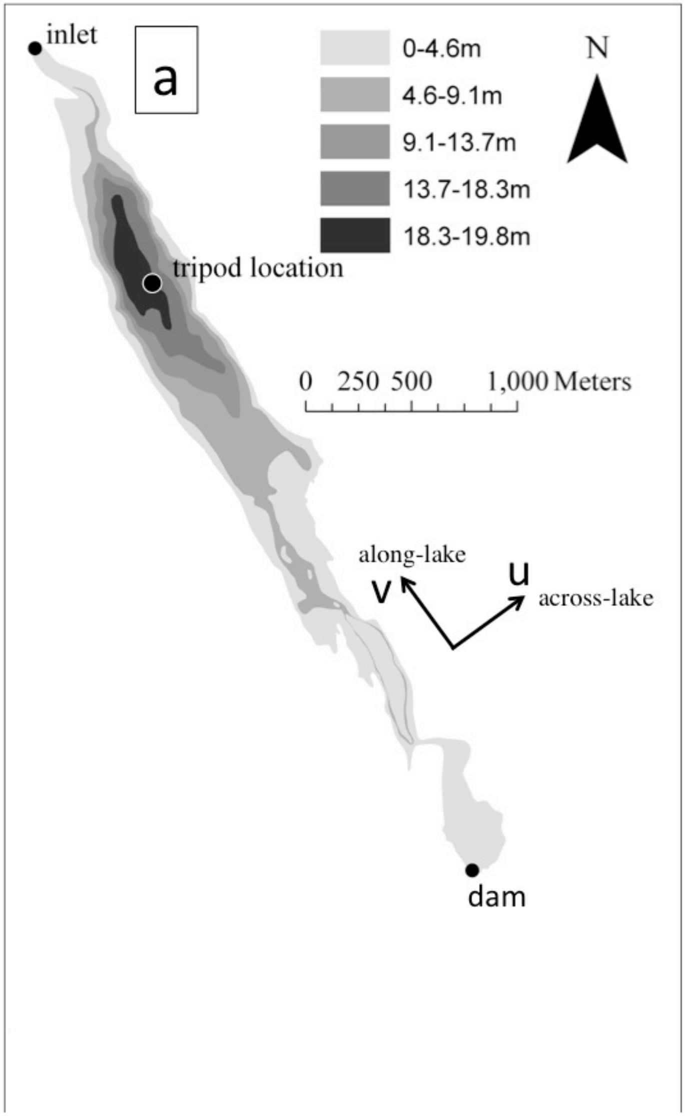
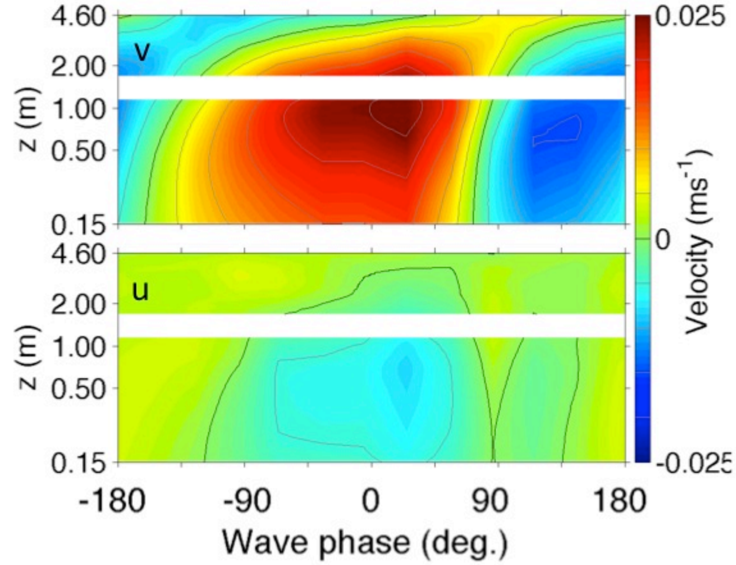
Outline

1. Instrumentation
2. Lakewide internal waves
3. Boundary layer, deep lake
4. Boundary layer, thermocline
5. Summary/discussion of periodic stratification

Phase-averaged flow

- Velocities dominantly along-lake, but deflected left in bottom boundary layer.





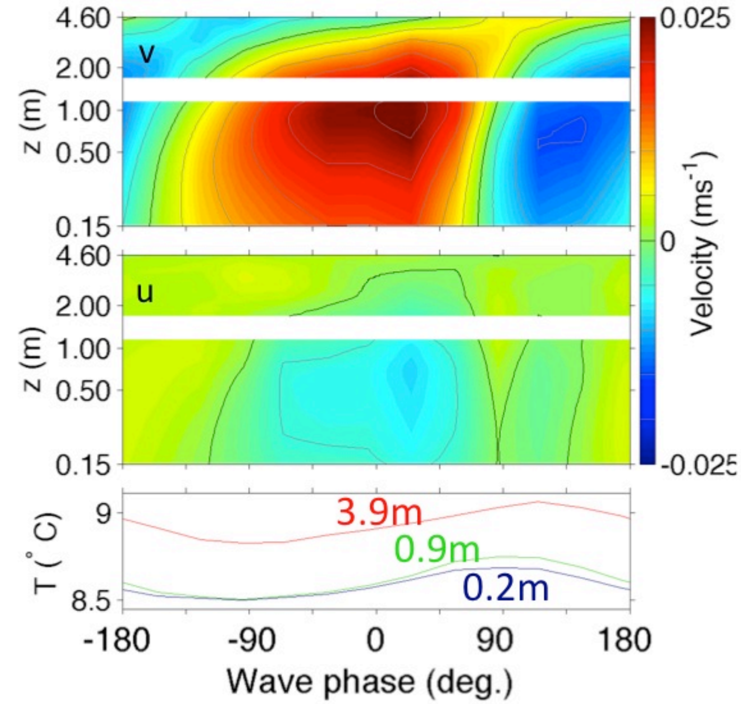
For several weeks of data, every time was associated with a waves phase, with phase varying 360 deg in each (roughly diurnal) wave cycle. Then all data with roughly the same phase averaged, to produce phase averages shown above.
Therefore, phase average shows something like the "average wave".

Note leftward deflection in bottom boundary layer - similar to bottom Eckman layer, but magnitude of deflection smaller than in classic Eckman Layer

Phase-averaged flow

- Velocities dominantly along-lake(v), but deflected left in bottom boundary layer.

- Temperature and stratification lag downslope flow (negative u) by ≈ 90^0.

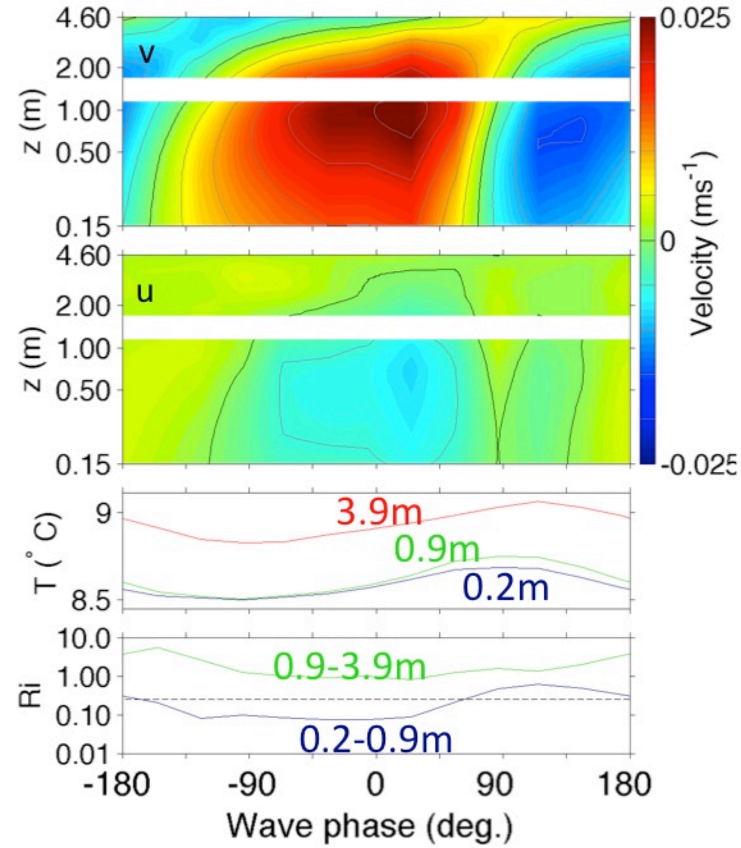


Phase-averaged flow

- Velocities dominantly along-lake(v), but deflected left in bottom boundary layer.

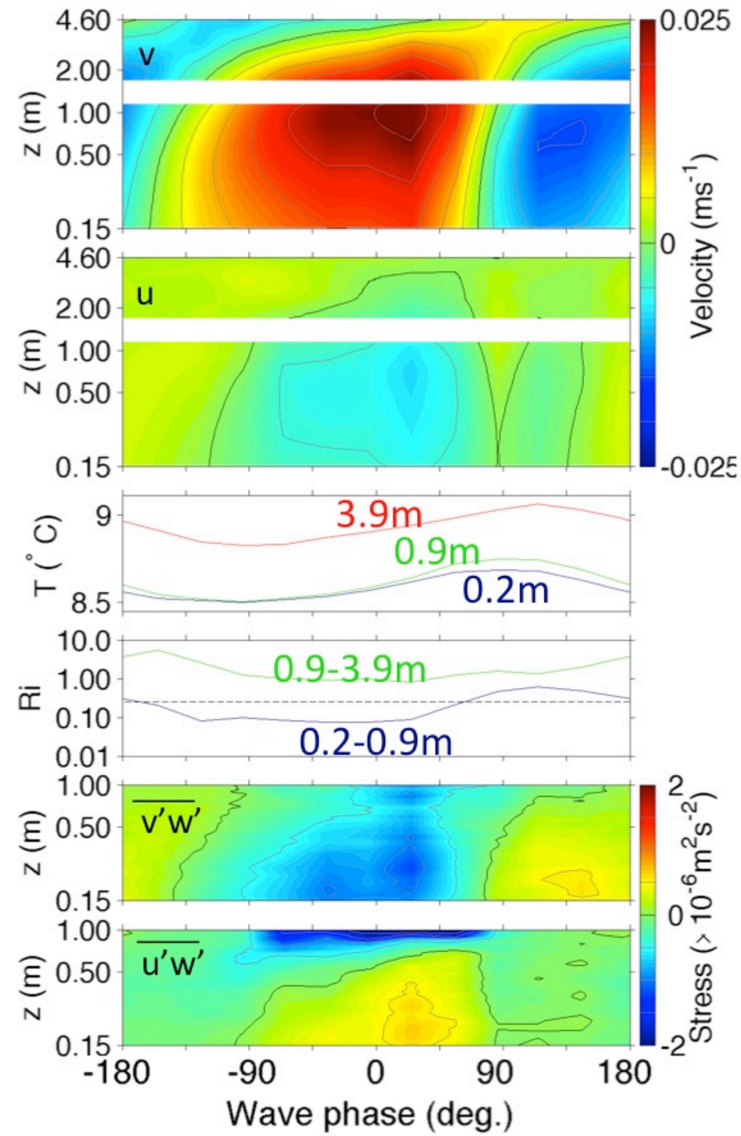
- Temperature and stratification lag downslope flow (negative u) by ≈ 90^0.

- Richardson numbers always >0.25 above boundary layer, intermittently >0.25 in boundary layer.



Phase-averaged flow

- Velocities dominantly along-lake(v), but deflected left in bottom boundary layer.



- Temperature and stratification lag downslope flow (negative u) by ≈ 90^0.
- Richardson numbers always >0.25 above boundary layer, intermittently >0.25 in boundary layer.
- Reynolds stress varied with elevation within 0.5 m of bed.

Turbulent Reynolds stress estimated by differencing along-beam velocity variances.

Phase-averaged flow

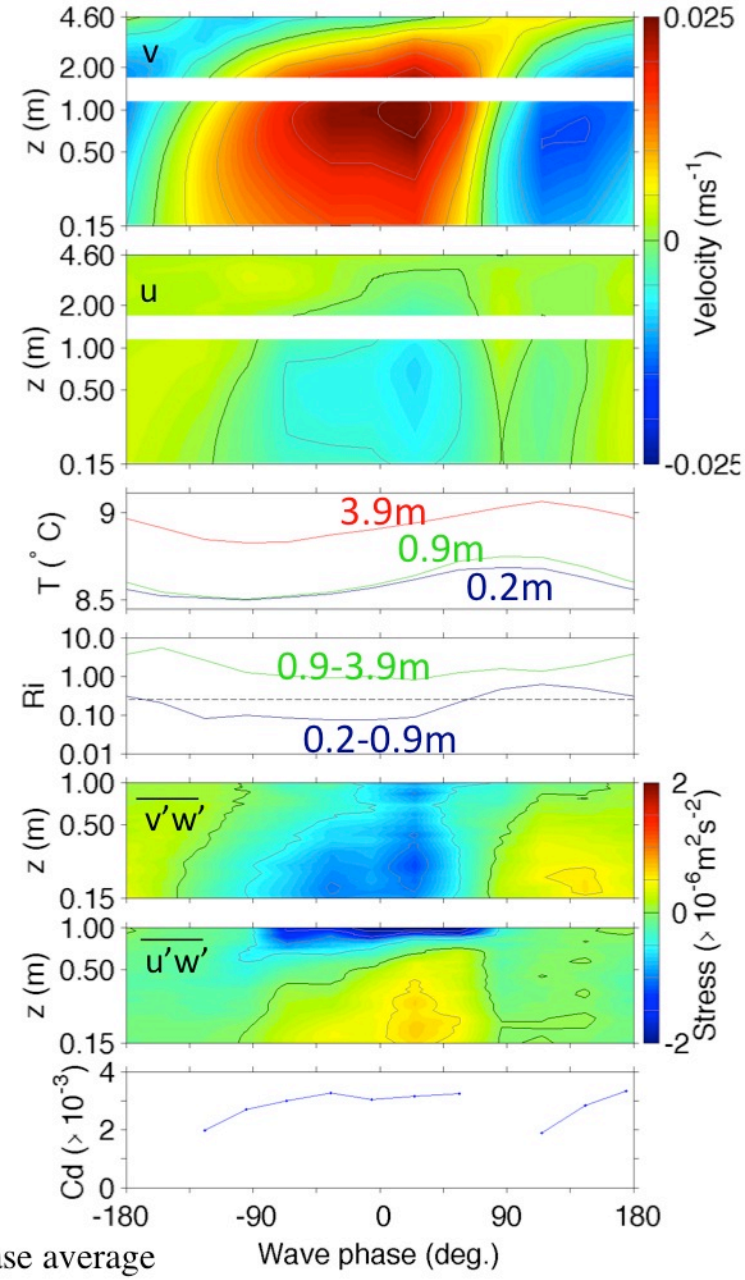
- Velocities dominantly along-lake(v), but deflected left in bottom boundary layer.

- Temperature and stratification lag downslope flow (negative u) by 90^0.
- Richardson numbers always >0.25 above boundary layer, intermittently >0.25 in boundary layer.
- Reynolds stress varied with elevation within 0.5 m of bed.

- Cd not strongly phase-dependent

Phase-averaged drag coefficient: $C_d = \mathrm{Re}\left\{ \frac{\langle \tau_x + i\tau_y \rangle}{\langle \|u+iv\| (u+iv) \rangle} \right\}$

$\langle \ \rangle$ = phase average



Buoyancy fluctuations

- Vertically integrated balance:

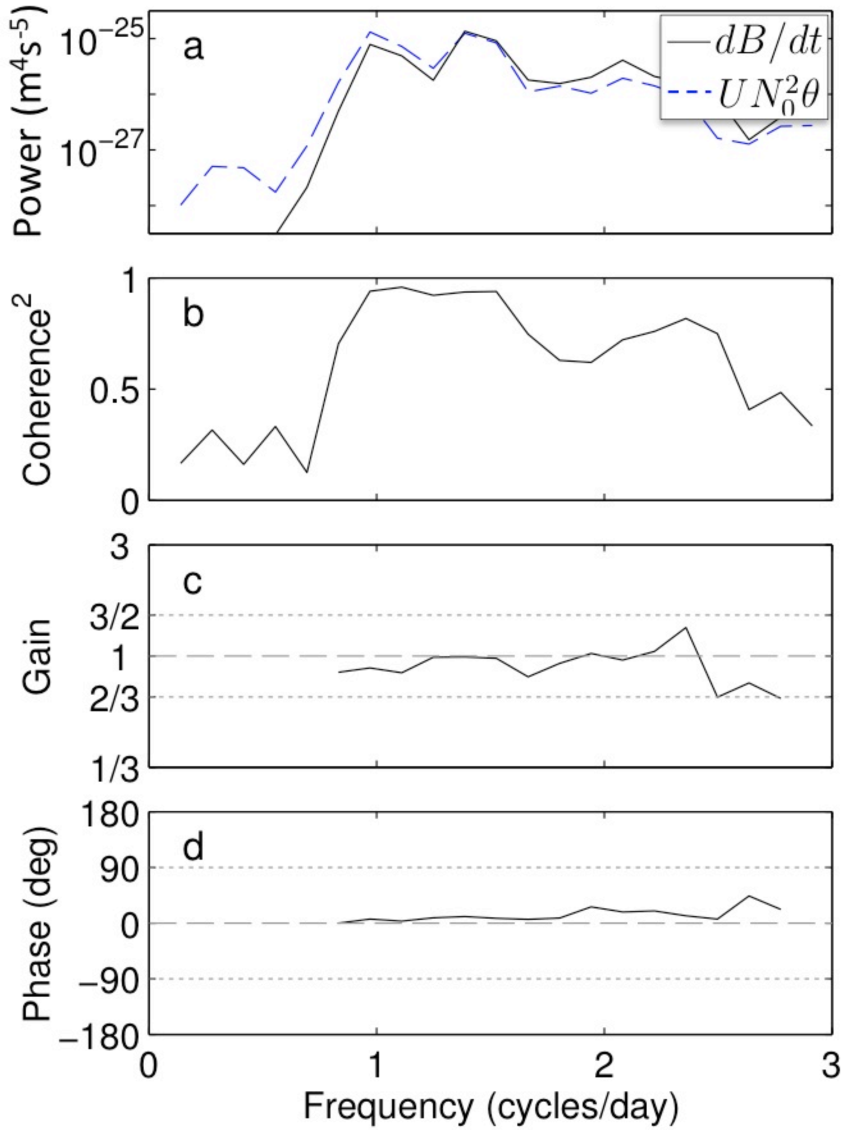
$$\frac{\partial B}{\partial t} + U N_0^2 \theta = 0,$$

$$b = (\rho_I - \rho) g / \rho_0$$

$$B = \int_0^{z_1} b \, dz.$$

$$U = \int_0^{z_1} u \, dz.$$

θ = slope = 0.03 (= 1.7^0)



Buoyancy fluctuations result from up/downslope advection of water

Along-lake velocity fluctuations

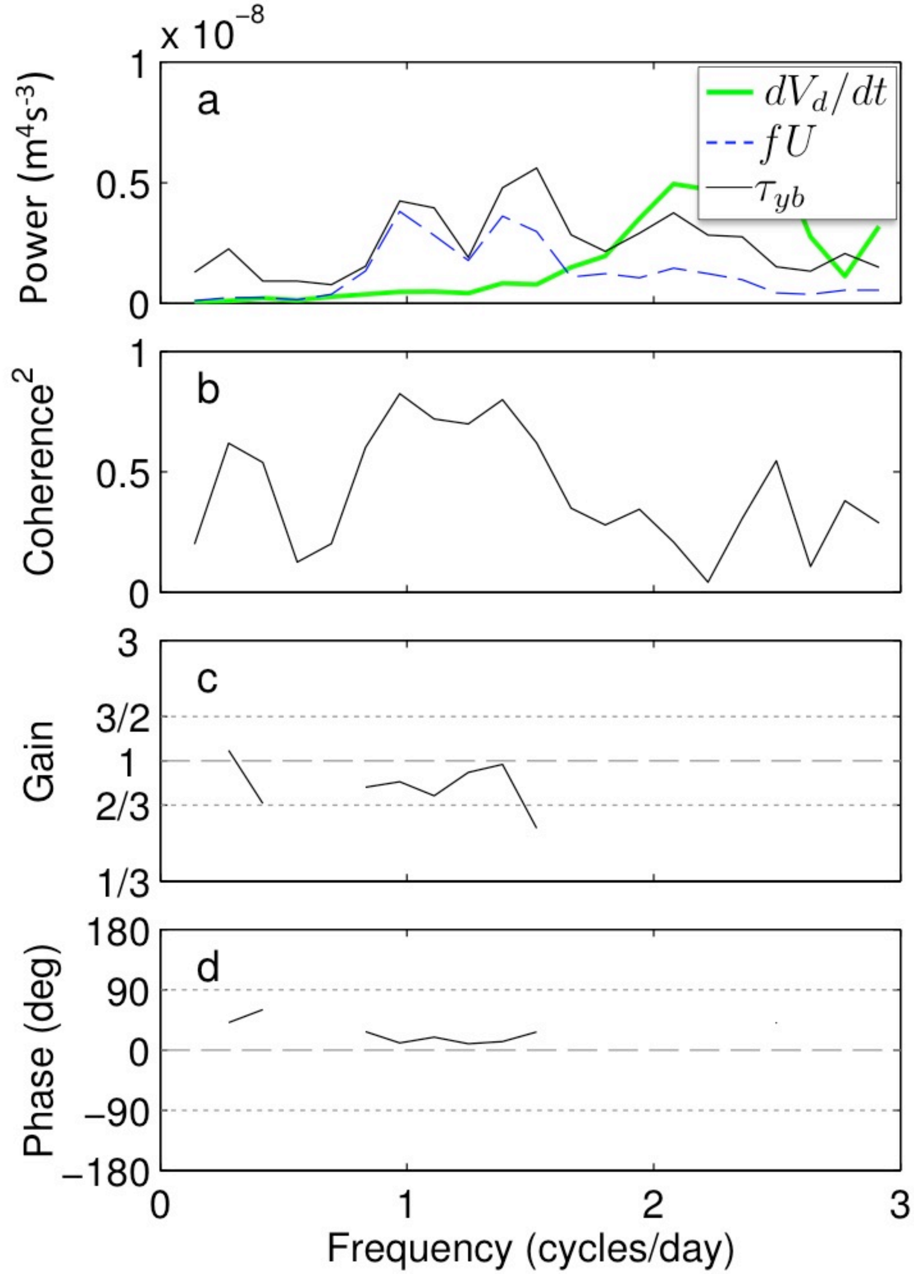
- Vertically integrated balance:

$$\frac{\partial V_d}{\partial t} + fU + \tau_{y,b} = 0,$$

$$V_d = \int_0^{z_1} (v - v_1)\, dz.$$

$$U = \int_0^{z_1} u\, dz.$$

$\tau_{y,b}$ = along-lake bed stress



At 1 cycle/day, along-lake balance is primarily between Coriolis and bed stress - like in classic Eckman layer

Across-lake velocity fluctuations

- Vertically integrated balance:

$$\frac{\partial U}{\partial t} - fV_d + B_d\theta + \tau_{x,b} = 0,$$

$$V_d = \int_0^{z_1} (v - v_1)\, dz.$$

$$U = \int_0^{z_1} u\, dz.$$

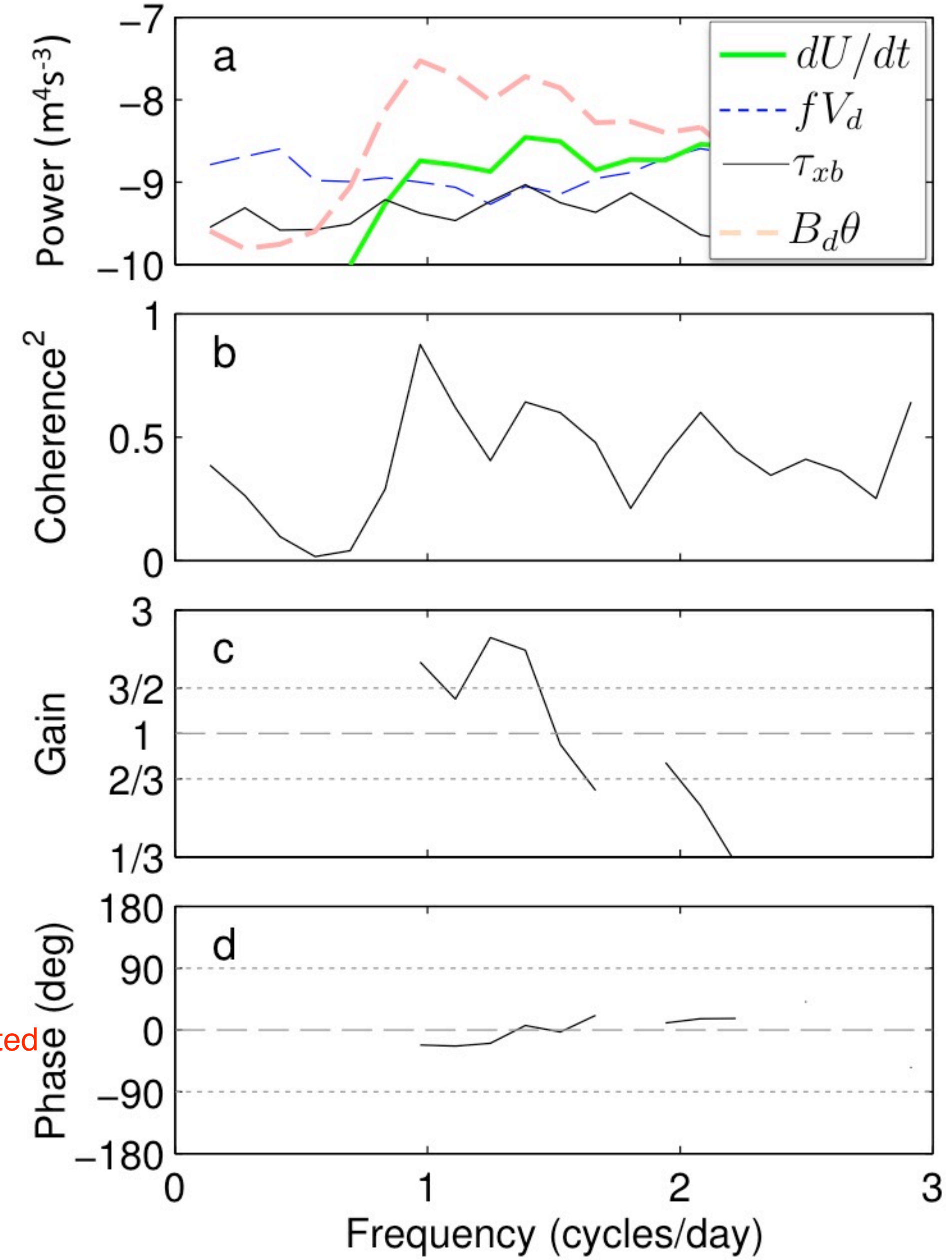
$$B_d = \int_0^{z_1} (b - b_1)\, dz.$$

θ = slope

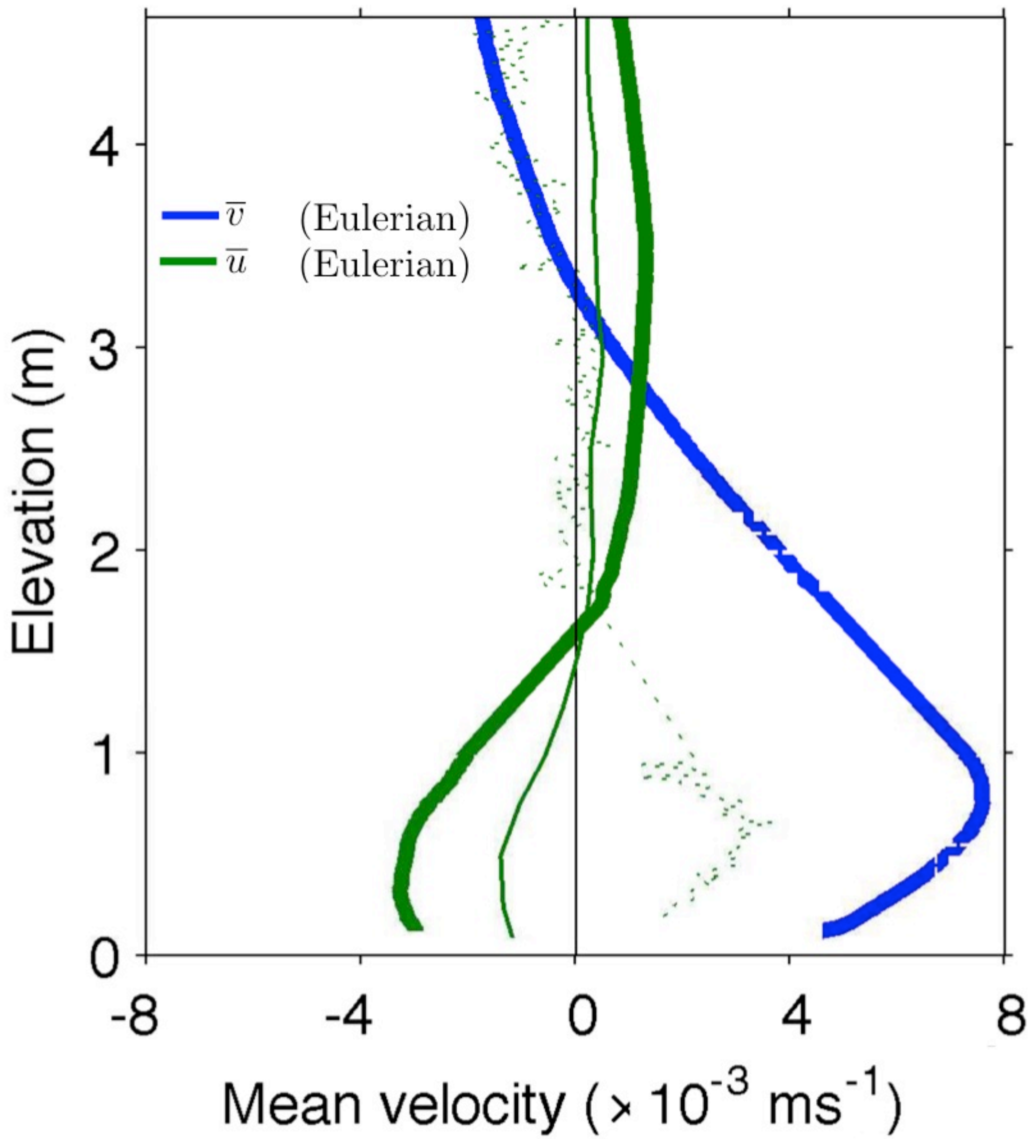
$\tau_{x,b}$ = across-lake bed stress

For x-lake flow, balance is not primarily between Coriolis and bed stress (classic Eckamn balance), but instead buoyancy is largest term, as expected given the large Buger number.

(Burger no = Nθ/f = 2.3)



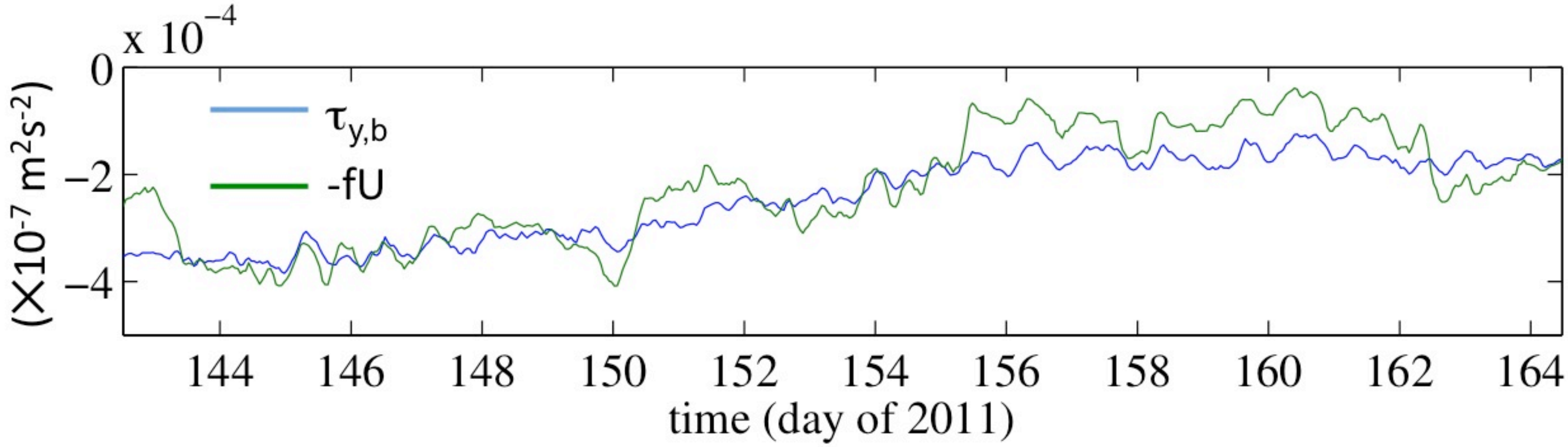
Eulerian Mean Flows



When daily fluctuations associated with waves are averaged out, residual mean is small, with along-lake & downslope boundary layer jet dominant

Wave-averaged along-lake force balance

$$\cancel{\frac{\partial V_d}{\partial t}} + fU + \tau_{y,b} = 0,$$



Along-lake momentum balance same as classic Eckman balance

Not an arrested Ekman layer

$$\cancel{\frac{\partial V_d}{\partial t}} + fU + \tau_{y,b} = 0,$$

$$\cancel{\frac{\partial B}{\partial t}} + UN_0^2\theta = 0,$$

Why doesn't $\bar{U} = 0$?

The standard theory says mean U should be (nearly) zero.

Stokes drift?

$$\frac{\partial B}{\partial t} + UN_0^2\theta = 0,$$

- Adding wave term $\int\left(\frac{\partial \overline{u_w b_w}}{\partial x} + \frac{\partial \overline{w_w b_w}}{\partial z}\right)dz$ ...

  ...introduces Stokes drift advection.

Stokes drift is difference between (Eulerian) mean velocity and mean particle velocity.
This difference arrises from presence of internal waves.

Stokes drift neglected in standard equations for sloping, stratified boundary layers

Stokes drift?

$$\frac{\partial B}{\partial t} + UN_0^2\theta = 0,$$

- Adding wave term $\int\left(\frac{\partial\overline{u_w b_w}}{\partial x} + \frac{\partial\overline{w_w b_w}}{\partial z}\right)dz$ …

  …introduces Stokes drift advection.

- Assume waves propagate without change of form.

- Stream function for Stokes drift:

  $\overline{u_w Z_w}$, where $Z_w = \int w_w \, dt$

Stokes drift?

$$\frac{\partial B}{\partial t} + U N_0^2 \theta = 0,$$

- Adding wave term $\int \left(\frac{\partial \overline{u_w b_w}}{\partial x} + \frac{\partial \overline{w_w b_w}}{\partial z} \right) dz$ …

  …introduces Stokes drift advection.

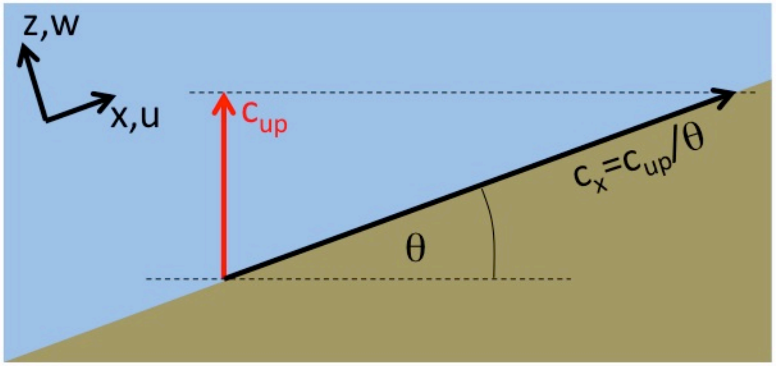
- Assume waves propagate without change of form.

- Stream function for Stokes drift:

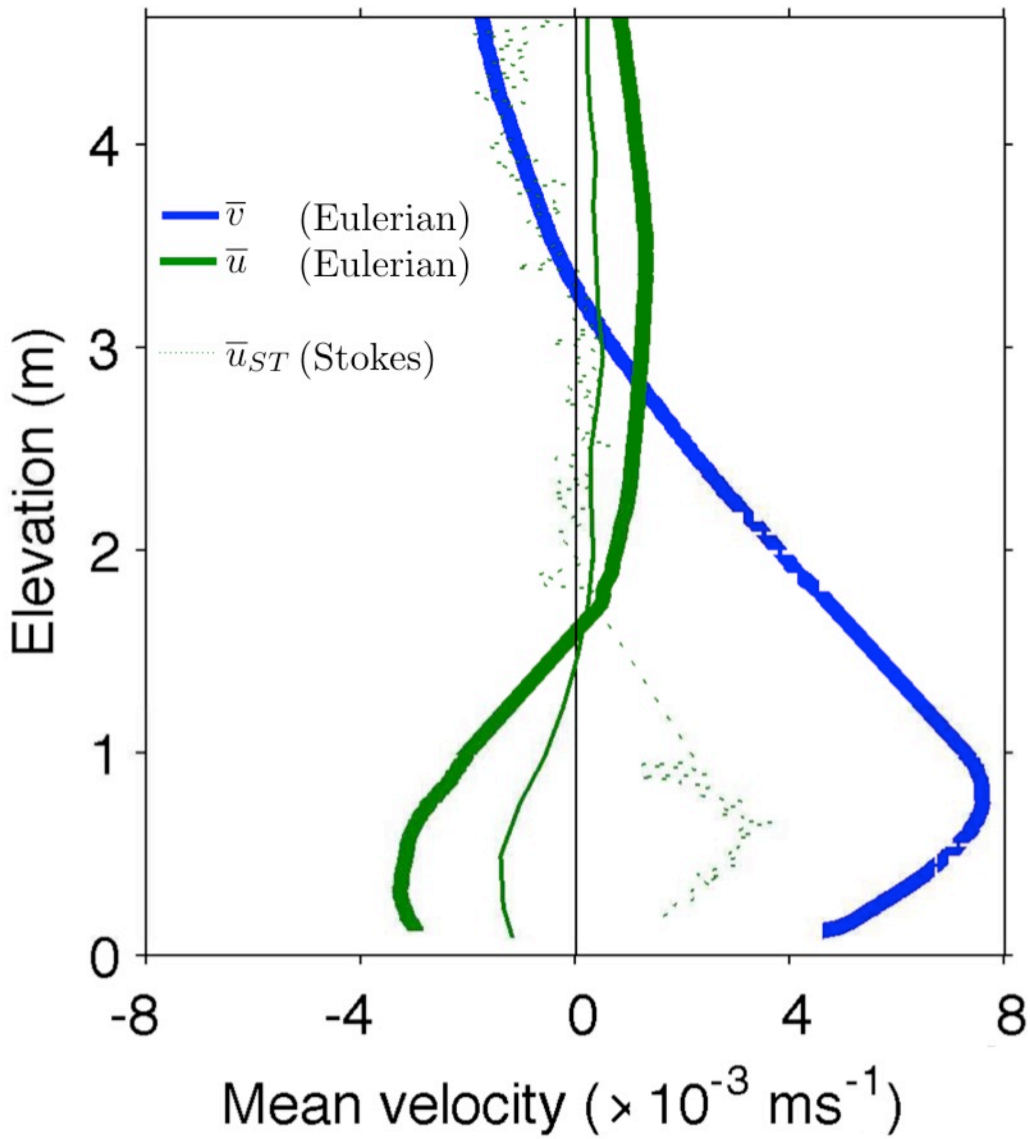
  $\overline{u_w Z_w}$, where $\mathrm{Z}_w = \int w_w \, dt$

- Boundary-normal velocity:

  $$w_w = -\frac{\partial}{\partial x} \int_0^z u_w \, dz' = \left(\frac{1}{c_x} \right) \frac{\partial}{\partial t} \int_0^z u_w \, dz'$$

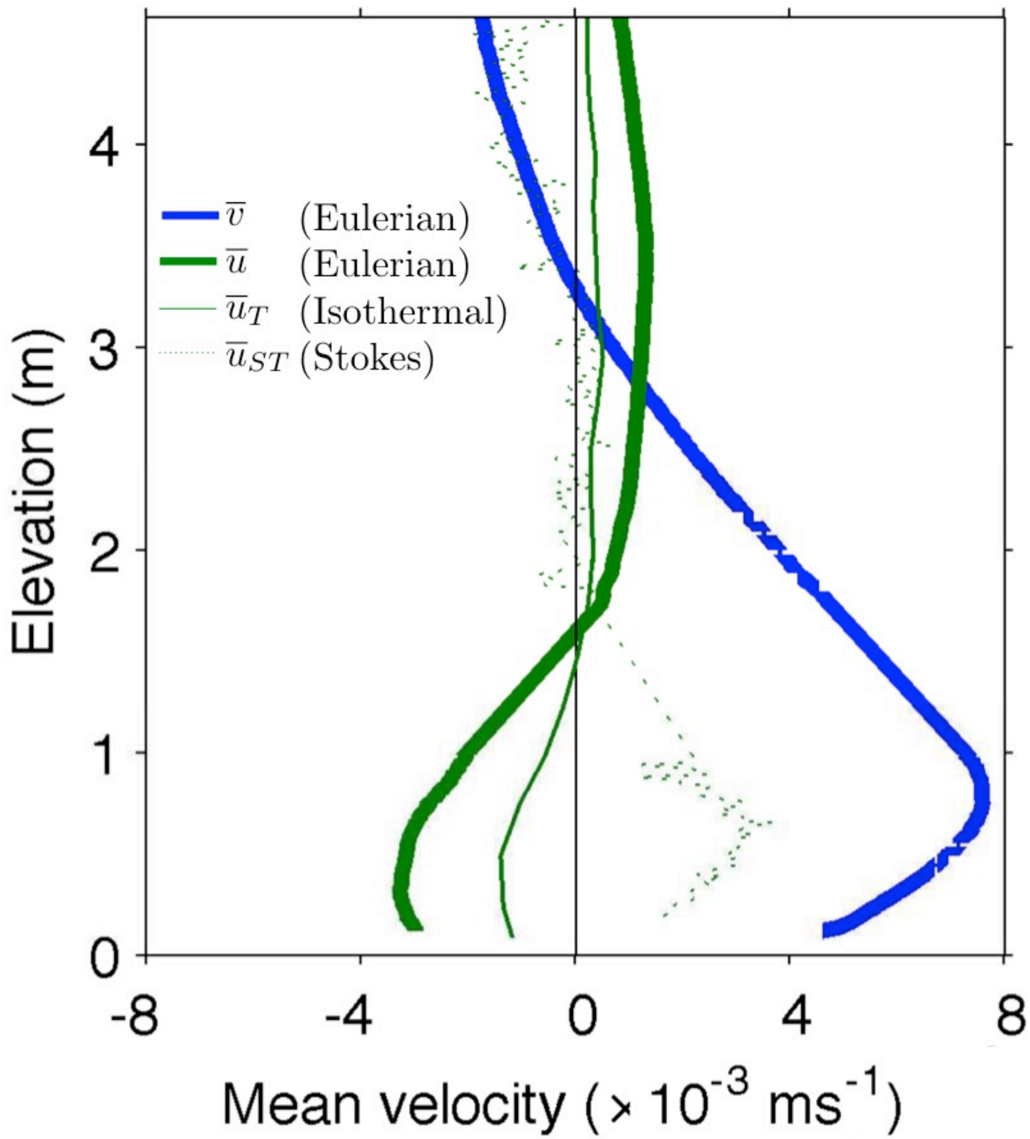


Estimated Stokes drift cancels Eulerian flow



So, despite downslope Eulerian mean velocity, particles might not on average be moving downslope

Estimated Stokes drift cancels Eulerian flow



Possible wave-driven flow?

- Hypothesis:

1. Internal waves transport dense water upslope.
2. gravitational force on dense water generates downslope Eulerian flow (arrest holds in Lagrangian-mean sense).
3. Coriolis acting on Eulerian downslope flow drives along-isobath flow.

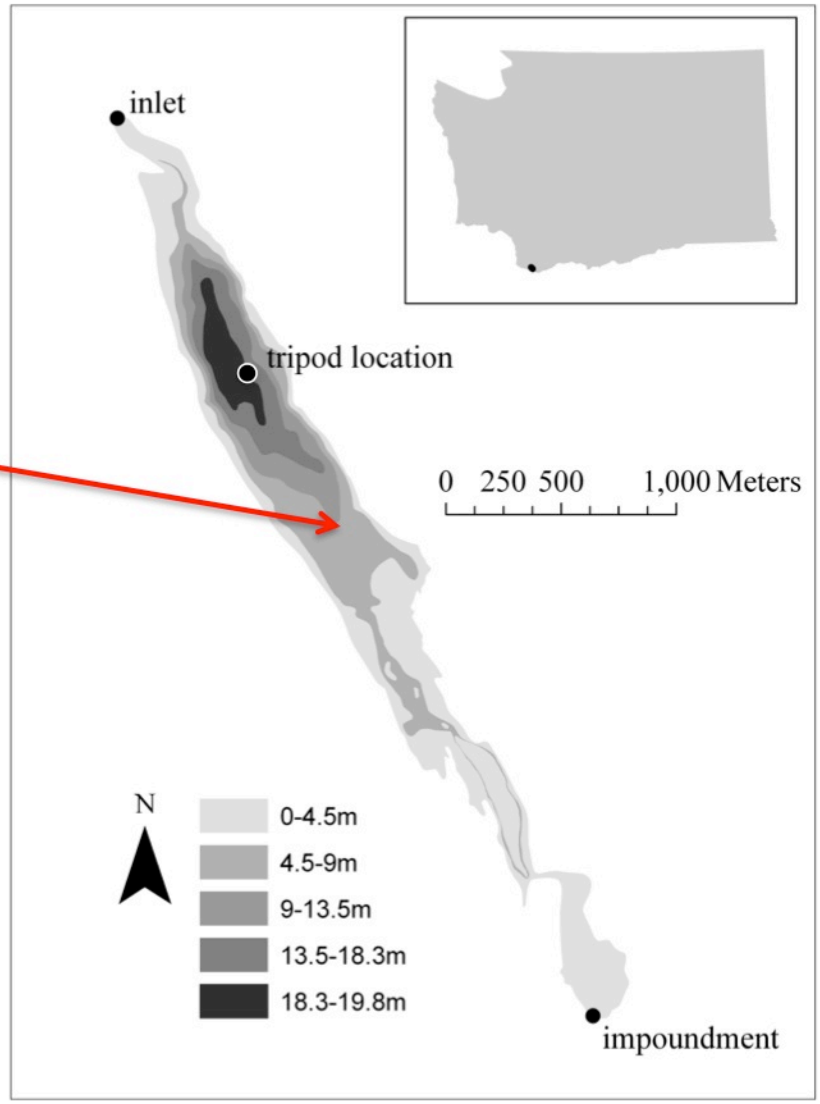
(wave momentum fluxes could modify picture)

Outline

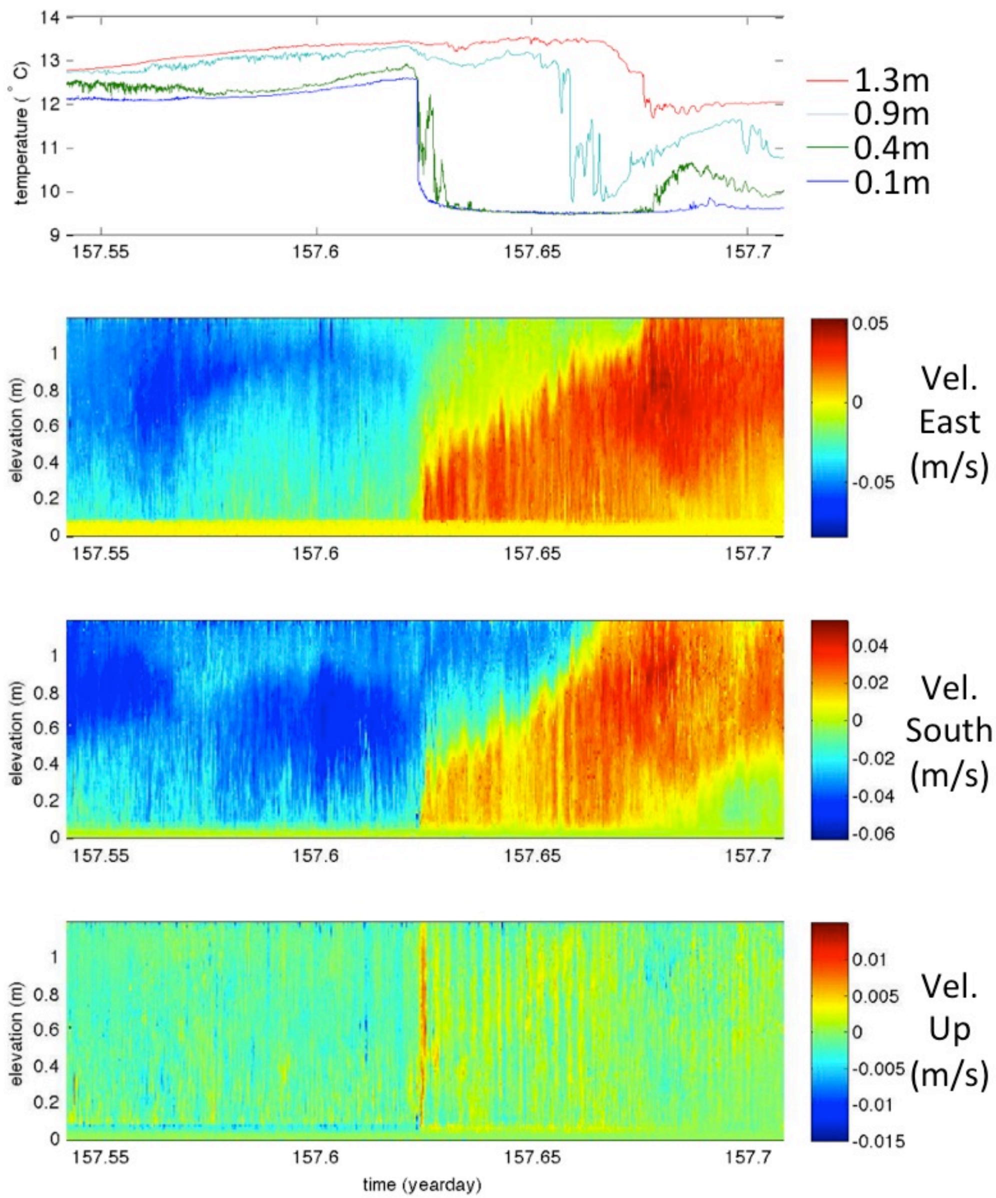
1. Instrumentation
2. Lakewide internal waves
3. Boundary layer, deep lake
4. Boundary layer, thermocline
5. Summary/discussion of periodic stratification

Full-depth velocity profiles

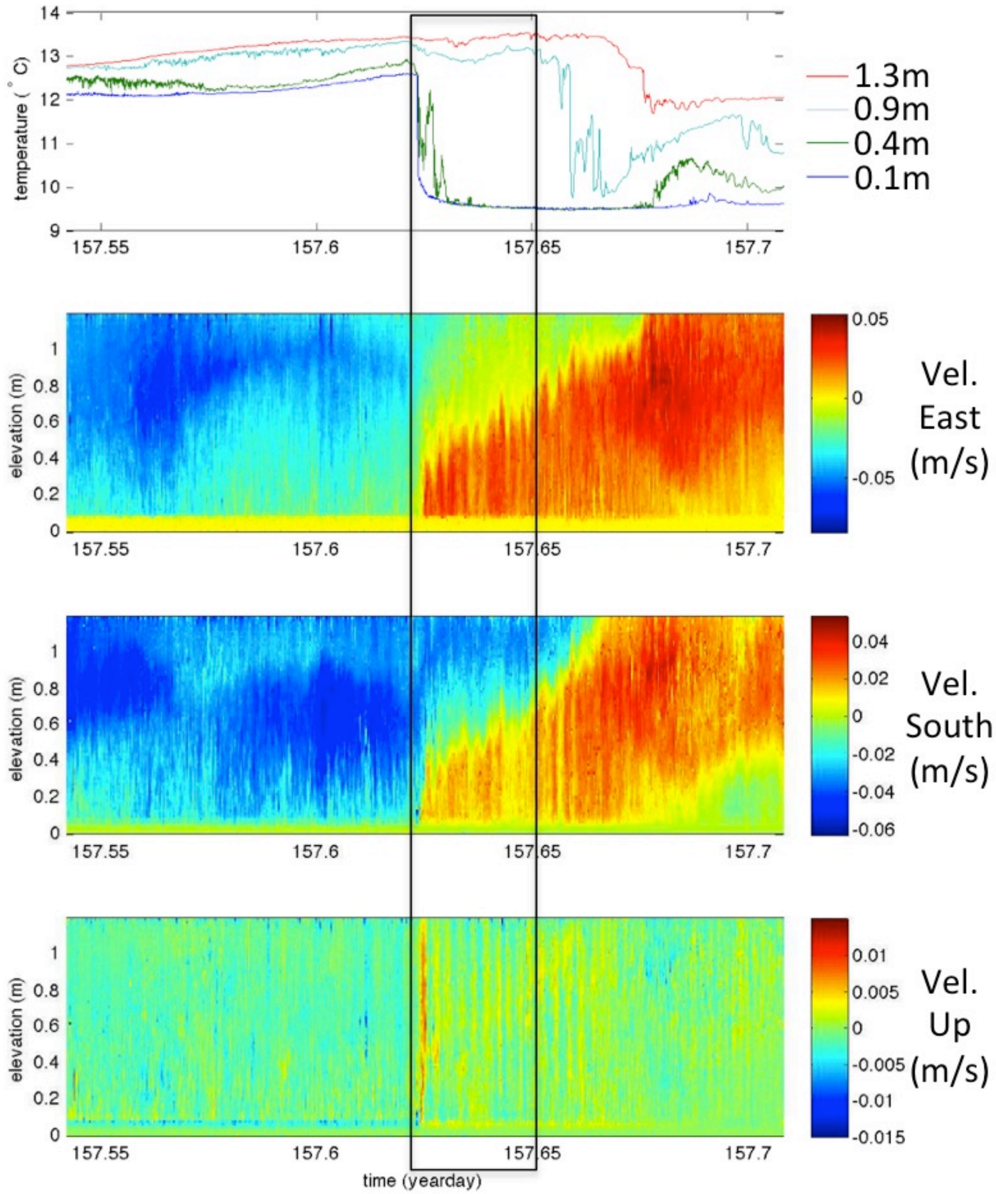
Tripod deployed where thermocline meets bed



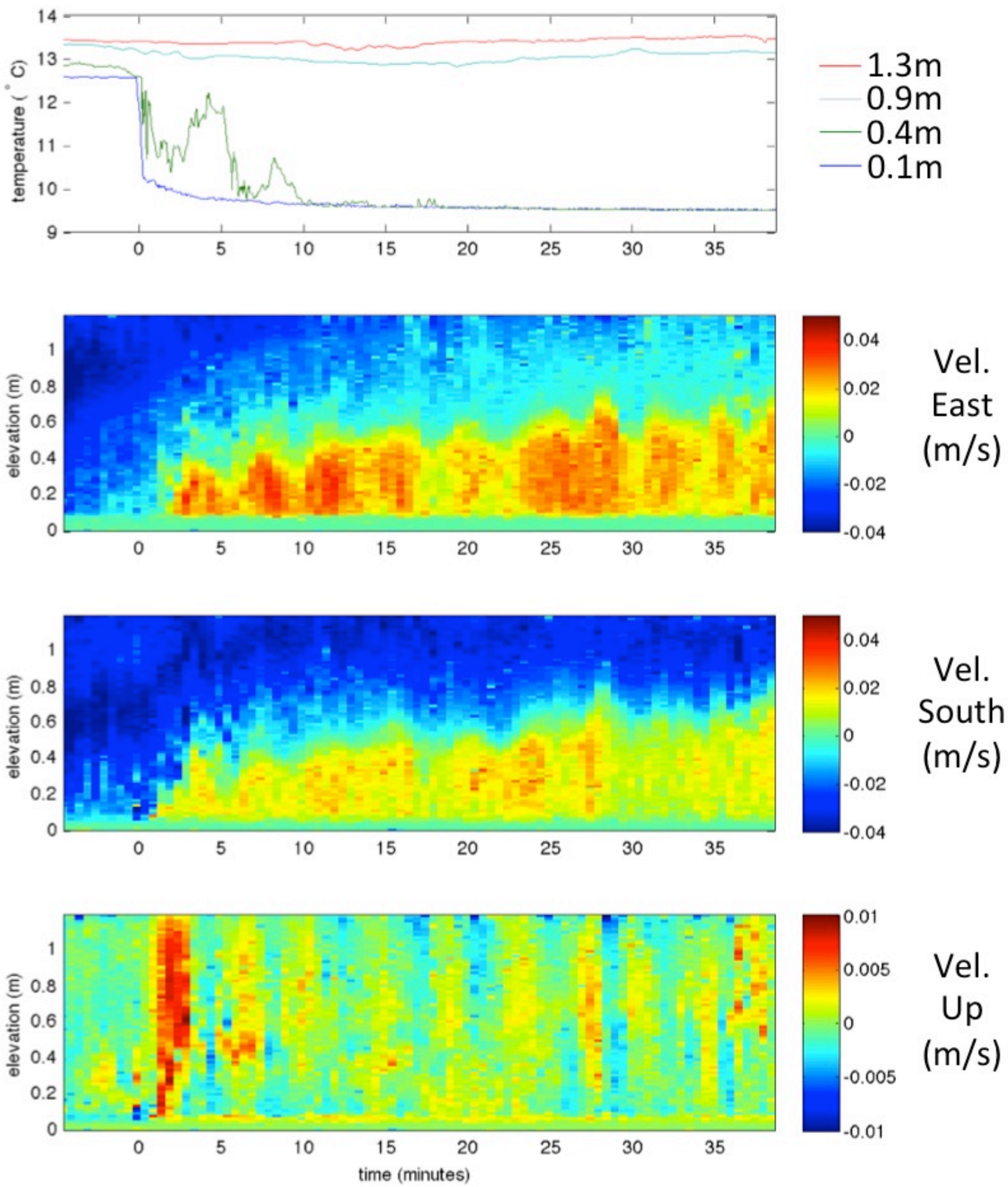
Sudden arrival of upslope flow



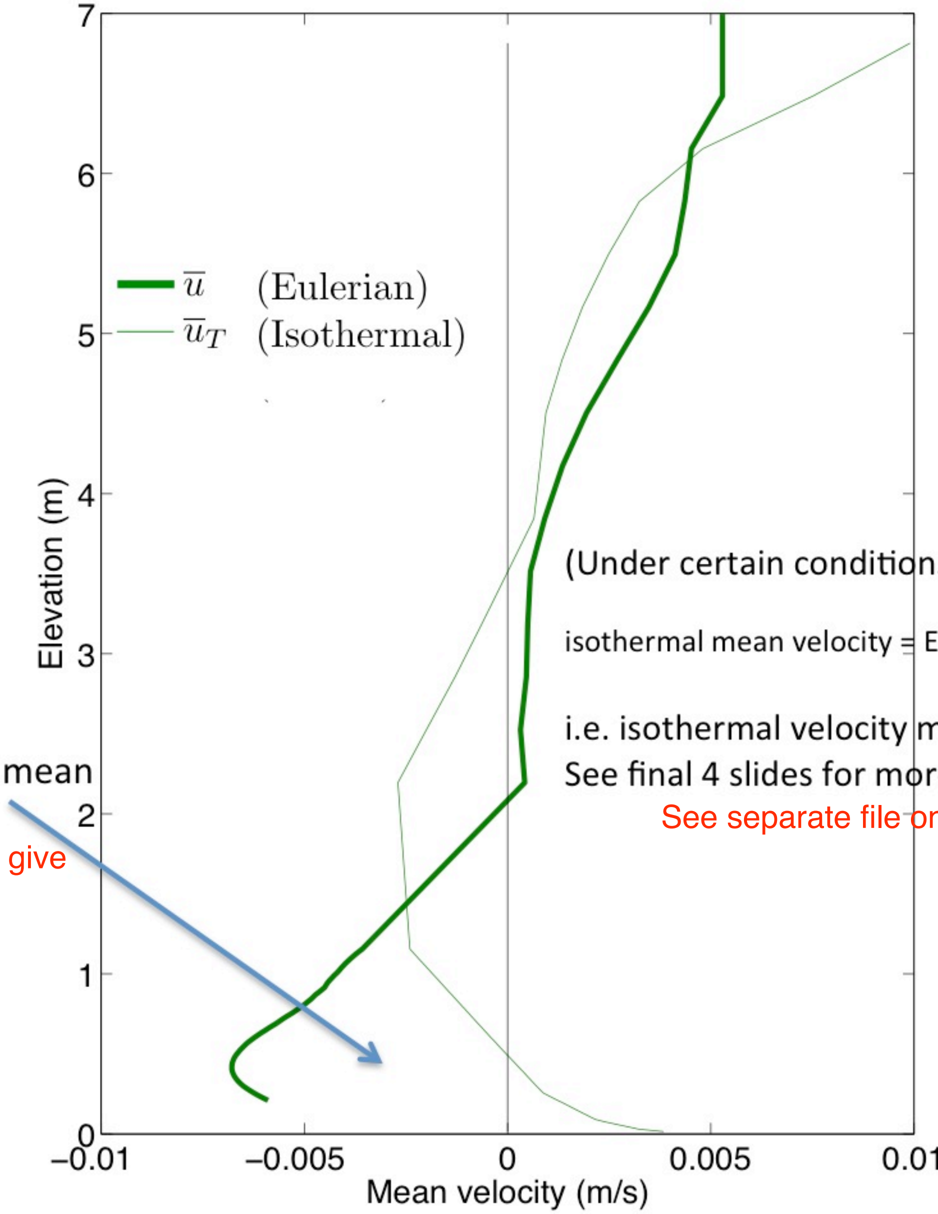
Sudden arrival of upslope flow



Undular bores



Mass transport



(Under certain conditions,

isothermal mean velocity = Eulerian mean velocity + Stokes drift,

i.e. isothermal velocity mean velocity of water particles.
See final 4 slides for more details.)

See separate file on velocities in isothermal coordinates

Particle transport < Eulerian mean

Here, up-slope Stokes drift is again canceling downslope Eulerian flow, to give small net transport

Outline

1. Instrumentation
2. Lakewide internal waves
3. Boundary layer, deep lake
4. Boundary layer, thermocline
5. Summary/discussion of periodic stratification

Summary

1. Lakewide internal waves propagate vertically owing to surface generation and lakebed dissipation.
2. Oscillating stratification observed, with destratification on upslope flow.
3. Mean flows strongest near boundary layer, downslope Eulerian flow not arrested.
4. Upslope mass transport by waves appears to roughly cancel Eulerian downslope flow.